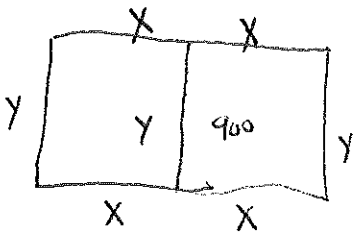
Calculus 1
Worksheet: Optimization (Cost Problems)

Name_____
Date_____

1) A farmer wants to fence off two side by side identical rectangular pens for his cows. Each pen is to contain 900 square feet. The outer fence requires a heavy-duty material that costs $2.00 per foot. The fence in the center is a lighter grade of fence and costs $1.00 per foot. What are the dimensions of the pen that will minimize the cost of the fence?



$E = 2(4x) + 2(2y) + 1(y)$

$E = 8x + 4y + y$

$E = 8x + 5y$

$A = x \cdot y$

$900 = x \cdot y$

$\frac{900}{x} = y$

$E = 8x + 5\left(\frac{900}{x}\right)$

$E = 8x + 4500x^{-1}$

$E' = 8 - \frac{4500}{x^2} = 0$

$8 = \frac{4500}{x^2}$

$8x^2 = 4500$

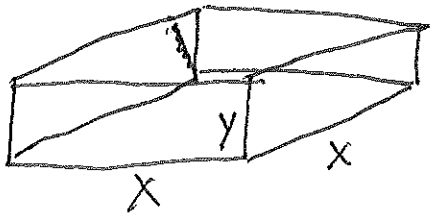
$x^2 = 562.5$

$x = 23.717$

$y = 37.947$

23.717 ft x 37.947 ft

2) A rectangular box with a square base and without a lid has to maintain a surface area of 1000 square feet. What are the dimensions of the box if the volume is maximized?



$SA = x^2 + 4xy$

$1000 = x^2 + 4xy$

$\frac{1000 - x^2}{4x} = y$

$y = 9.129$

$V = x \cdot x \cdot y$

$V = x^2\left(\frac{1000 - x^2}{4x}\right)$

$V = 250x - \frac{1}{4}x^3$

$V' = 250 - \frac{3}{4}x^2 = 0$

$250 = \frac{3}{4}x^2$

$1000 = 3x^2$

$\frac{1000}{3} = x^2$

$x = 18.257$

18.257 ft x 18.257 ft x 9.129 ft